

(Time : 3 hours)

Note: - (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q 2 to Q 6.

(3) Figures to the right indicate full marks.

1(a) Evaluate $\int_0^{1+i} (x^2 + iy) dz$, along the parabola $y = x^2$. (5)1(b) Find a vector orthogonal to both $u = (1, 1, 0)$, $v = (0, 1, 1)$. (5)

1(c) Calculate the coefficient of correlation between X and Y

X	3	5	4	6	2
Y	3	4	5	2	6

(5)

1(d) The Probability function of a random variable X is given by (5)

X	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

Find (i) k (ii) mean (iii) variance

2(a) Find the extremals of $\int_0^1 (xy + y^2 - 2y^2y') dx$. (6)

(b) Monthly salary X in a big organization is normally distributed with mean Rs. 3000 and standard deviation of Rs. 250. What should be the minimum salary of a worker in this organization, so that the probability that he belongs to top 5% workers? (6)

(c) Find all possible Laurent's expansions of the function $f(z) = \frac{2}{(z-1)(z-2)}$. (8)

3(a) A random variable X has the following density function

$$f(x) = \begin{cases} \frac{x}{6} + k, & x > 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

find k and $P(1 \leq x \leq 2)$. (6)

(b) . Evaluate

$$\int_C \frac{\sin^6 z}{(z-\frac{\pi}{2})^3} dz, \text{ where } C \text{ is the circle } |z| = 2. (6)$$

(c) Reduce the quadratic form $x^2 + 2y^2 + 2z^2 - 2xy - 2yz + zx$ to normal form through congruent transformations. Also find its rank, signature and value class. (8)

4(a) Construct an orthonormal basis of R^3 using Gram-Schmidt process to

$$S = \{(3, 0, 4), (-1, 0, 7), (2, 9, 11)\}.$$

(b) Find the Probability that at most 5 defective diodes will be found in pack of 600 diodes ; if previous data shows that 3 % of each such diodes are defective. (6)

(c) Solve by Rayleigh-Ritz method the boundary value problem (8)

$$I = \int_0^1 (2xy - y^2 - y'^2) dx \text{ given } y(0) = 0 \text{ and } y(1) = 0.$$

5(a) Find the spearman's rank of correlation for the following data

$$\begin{array}{l} X : 12 \quad 17 \quad 22 \quad 27 \quad 32 \\ Y : 113 \quad 119 \quad 117 \quad 115 \quad 121 \end{array} \quad (6)$$

(b) Using Cauchy's Residue theorem evaluate $\oint_C \frac{e^{2z}}{(z-\pi i)^3} dz$ where C is $|z - 2i| = 4$. (6)

(c) Find the singular value decomposition of the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ (8)

6(a) Verify Cauchy-Schwartz Inequality for the vectors

$$u = (-4, 2, 1) \text{ and } v = (8, -4, -2). \quad (6)$$

(b) Determine whether the set of vectors of the form (a, b, c) where $b = a + c$ form a subspace of R^3 under usual addition and scalar multiplication. (6)

(c) Obtain the equations of the line of regression for the following data.

$$\begin{array}{l} X : 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \\ Y : 11 \quad 14 \quad 14 \quad 15 \quad 12 \quad 17 \quad 16 \end{array}$$

(8)